Proposal of a Three Levels Output Space Mapping Strategy

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Abstract — Optimal design with finite element model is often expensive in terms of computation time. The space-mapping technique allows benefiting both the rapidity of the analytical model and the accuracy of the finite element model. In this paper, based on a surrogate model, a 2D FE model and a 3D FE model, a three levels adapted output space-mapping technique is proposed. The results show that the proposed algorithm allows saving of computation time compared to the classical two levels output space-mapping.

I. Introduction

For design and analysis of a linear electrical motor, an analytical solution of electrical and magnetic fields is difficult to achieve, due to the end and edge effects, but also the non-linearity of the phenomena. The numerical methods such as finite element method allow overcoming these difficulties [1]. However optimal design of an electrical device using finite element models (FEM) is complex and time consuming. The high computation time, numerical noise and the mesh quality of the FEM make it difficult to use in an optimization process.

Several variants of space-mapping technique [2] are recently being used in solving optimization problems of electromagnetic devices [3], which allow benefiting the rapidity of the analytical model and the accuracy of the FEM by aligning the both models. Space-mapping technique converges faster by avoiding the use of the FEM during the optimization process.

In this paper, three linear induction motor (LIM) models with different accuracies (coarse, medium and fine) are presented and the optimization problem is introduced. Secondly, the three levels adapted output space-mapping technique (OSM) is described and applied to the three models. Thirdly, the method efficiency is analyzed and compared to the classical two levels OSM.

II. Optimization Problem

A. Optimization Problem Formulation

The device to be studied is a double-sided LIM. It consists of two symmetrical primaries placed face-to-face and a secondary placed between the two primaries achieved by an aluminum plate. The primaries have three concentrated windings and are fed using a three-phase AC voltage. Fig. 1 shows the 3D FE model of the LIM.

A single objective optimization problem is set up for the optimal sizing of the double-sided LIM. It consists of four design variables, and among them there are three geometrical variables: $tw1$, $tw2$, $tw3$ for the width of the motor teeth, and U for the fed voltage of the primary. The geometrical design variables are shown on Fig. 1. There are three constraints in this problem. The mass and the losses of the device should be respectively less than 2kg and 100W. The non-balance of the currents should be less than 10%. The objective function is to maximize the thrust force provided by the slip field and the induced current in the aluminum plate.

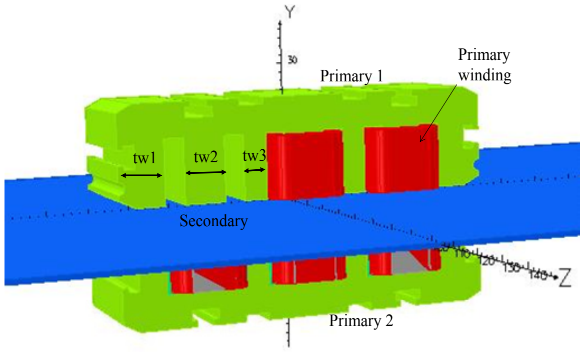

Fig. 1. 3D FEM of the LIM with two primaries and one aluminum plate

The optimization problem of the double-sided LIM is expressed in (1):

$$\begin{aligned}
&\min(-F)\\
&mass < 2kg,\ losses < 100W,\\
&\left|\frac{I_1}{I_2}-1\right| + \left|\frac{I_3}{I_2}-1\right| < 0.1 \qquad (1)\\
&tw1 \in [5,20],\ tw2 \in [3,20],\ tw3 \in [3,20],\\
&U \in [0,20]
\end{aligned}$$

B. Coarse, Medium, and Fine Models

The behavior of the double-sided LIM is studied by a surrogate (coarse) model, a 2D FEM (medium model) and a 3D FEM (fine model).

TABLE I - ERROR OF MODELS

Model type	*ΔLosses (%)*	*ΔF (%)*	*ΔEq (%)*	*Comp. time (s)*
Coarse	37.55	37.79	64.8	0.2
Medium	33.11	31.92	51.53	311
Fine	28.73	8.44	47.60	5400

Table I presents the error of the three models compared to the measurement and computing time of each model for the initial solution.

III. Three Levels Adapted Output Space-Mapping

Output space-mapping technique is investigated in order to obtain satisfactory results with a minimum number of computationally expensive 3D FEM evaluations [4]-[5]. It aims to use both the coarse and fine models to reduce the computation time and increase the accuracy of the obtained solution. The problem of the two levels OSM technique is its computing time when one 3D FEM evaluation is computationally expensive and the iterations number is important. To overcome this problem, a model with a medium accuracy is added between the coarse and fine models within the OSM algorithm. The medium model has rather good accuracy and smaller computing time compared to the fine model.

In general, the coarse computationally cheaper model is denoted by $c(z) \in \Re^m$ with $z \in Z \subset \Re^n$, the fine computationally expensive model is denoted by $f(x) \in \Re^m$ with $x \in X \subset \Re^n$, and the medium-accuracy model is denoted by $m(x) \in \Re^m$. In this case, the inputs of the three models are the same, i.e. $z \equiv x \subset \Re^n$. The nonlinear constraints of the coarse, medium, and fine models are $g_c(x)$, $g_m(x)$ and $g_f(x)$, respectively. The strategy of the three levels OSM consists initially of aligning the coarse model and the medium model by the corrective coefficients $\theta \in \Theta \subset \Re^m$. These coefficients are updated at each iteration to minimize the discrepancy between both models.

$$\begin{bmatrix} c(x,\theta_j) \\ g_c(x,\theta_j) \end{bmatrix} = diag(\theta_j) \cdot \begin{bmatrix} c(x) \\ g_c(x) \end{bmatrix} \tag{2}$$

$$\theta_{j+1} = \begin{bmatrix} m(x_j,\beta_j) / c(x_j) \\ g_m(x_j,\beta_j) / g_c(x_j) \end{bmatrix} \tag{3}$$

These coefficients are introduced into the coarse model to compute a new solution x_j for the next iteration. β is used to correct the medium model as explained later.

$$x_j = \arg\min_{x \in X} \left\| c(x,\theta_j) - y \right\| \text{ s.t. } g_c(x,\theta_j) \le 0 \tag{4}$$

where $y \in \Re^m$ denotes a vector of design specification and can be zeros in the case of minimization. The space-mapping between the coarse and medium models stops when (5) is checked,

$$\left\| \begin{bmatrix} c(x_j,\theta_j) \\ g_c(x_j,\theta_j) \end{bmatrix} - \begin{bmatrix} m(x_j,\beta_j) \\ g_m(x_j,\beta_j) \end{bmatrix} \right\| \le \varepsilon \tag{5}$$

where ε is the required accuracy and β_j is constant during this part of the algorithm. The second part of the algorithm consists of calculating the outputs of the fine model with x_j and aligning the medium model with the fine one using a vector of correctors β as:

$$\beta_{j+1} = \begin{bmatrix} f(x_j) / m(x_j) \\ g_f(x_j) / g_m(x_j) \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} m(x,\beta_j) \\ g_m(x,\beta_j) \end{bmatrix} = diag(\beta_j) \cdot \begin{bmatrix} m(x) \\ g_m(x) \end{bmatrix} \tag{7}$$

The three levels OSM algorithm stops when the discrepancy between the corrected medium model and the fine model is small enough:

$$\left\| \begin{bmatrix} m(x_j,\beta_j) \\ g_m(x_j,\beta_j) \end{bmatrix} - \begin{bmatrix} f(x_j) \\ g_f(x_j) \end{bmatrix} \right\| \le \varepsilon \tag{8}$$

To summarize, the three levels OSM algorithm carries out the following main steps:

0. Initialization $j = 0, \beta_o, \theta_o = I$
1. Optimization with the coarse corrected model: find the solution of (4), i.e. x_j
2. Evaluation of x_j with the medium corrected model, i.e. compute $m(x_j,\beta_j)$ and $g_m(x_j,\beta_j)$
3. Computation of the coarse model correctors for the next iteration with (3), i.e. θ_{j+1} and $\beta_{j+1} = \beta_j$
4. Until (5): $j = j+1$, go to step 1.
5. Evaluation of x_j with the fine model, i.e. compute $f(x_j)$ and $g_f(x_j)$
6. Computation of the medium model correctors for the next iteration with (6) , i.e. β_{j+1} and $\theta_{j+1} = \theta_j$
7. Until (8), $j = j+1$, go to step 1.
8. Stop

IV. Results And Conclusion

Both two and three levels OSM algorithms converge to the same solution. However, an important decrease of computation time is obtained thanks to the addition of an intermediate model. Table I shows that the number of 3D FEM evaluations is 4 times smaller and the overall optimization time, including the medium model evaluations, is 3 times smaller compared to the classical two levels OSM.

TABLE I - OSM RESULTS

OSM	3D FEM evaluations	medium model evaluations	time (s)
Three levels	2	10	15321
Two levels	8	-	43213

V. References

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